Journal of Statistical Physics, Vol. 78, Nos. 1/2, 1995

Acoustic Gas Slip Induced by Surface Waves

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Received September 13, 1993; final August 10, 1994

A boundary kinetic effect has been predicted on the basis of a model of interaction between Rayleigh surface acoustic waves (SAW) and a gas. The effect resembles some classic boundary effects such as thermal or diffusion gas slip. The functional dependence of the effect on gas and SAW parameters is presented. The slip phenomenon takes place due to angular restrictions of the scattered gas molecules because of the deformation of the solid surface.

KEY WORDS: Surface acoustic waves; acoustic flow; acoustic slip.

Traditionally, two aspects of the gas–surface acoustic wave (SAW) interaction problem are emphasized, the SAW attenuation and the flow induction. The problem of the SAW-induced flow is relatively new, but some research has been devoted to it. Aleksandrov *et al.*[1] an investigation of a compressible continuum[2] flow in a plane infinite channel due to SAW propagation. According to the calculation,[1] the flow velocity $\langle V \rangle$ of the gas is proportional to the second power of the SAW displacement velocity $a\omega$, as has also been experimentally confirmed (a is the SAW amplitude and ω the circular frequency). Theoretical models are given in refs. 2–4 for gas scattering on the surface perturbed by the wave in a free molecular regime. Unfortunately, there are no reliable experimental data concerning this regime and the theories disagree with each other. The first uses the phonon model,[2] where a phonon stream represents the SAW. The second theory relies upon the continuum model, where the SAW is supposed as a deformation of the solid continuum.[3,4] The theories agree with each other with

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[2] Here and below the terms "continuum" and "free molecular" are used in conjunction with the Knudsen number $Kn = l/\lambda$, where λ is the wavelength, l is the mean free path length in the gas.

0022-4715/95/0100-0161$07.50/0

respect to the entrainment velocity being proportional to the square of the SAW amplitude ($\langle V \rangle \propto a^2$). However, the frequency dependence is different, namely $\langle V \rangle \propto \omega^{3/2}$ for the phonon model and $\langle V \rangle \propto \omega^2$ for the continuum model. Such a discrepancy is strange, as there is a frequency range where both approaches are valid and therefore should lead to identical results.

In contrast to the above models, the present study considers a different mechanism of momentum transmission from SAW to gas molecules. It takes into account the angular restriction for the exit trajectories of the scattered molecules. Angular cutoffs have been considered in the ref. 3, but for the incident particles only. The restrictions arise due to the solid surface deformation by the SAW. This causes a gas drift, whose velocity is proportional to the first power of $a\omega$ and hence to the ratio of amplitude to wavelength; since always $a/\lambda \ll 1$, this effect is expected to dominate.

This paper presents the model of SAW–gas interaction in the free molecular regime ($l/\lambda \gg 1$). The model supposes that the solid is a continuum, the gas monoatomic, the gas–surface accommodation perfect, and that both parts of the system have the same temperature. The gas distribution function at infinity belongs to an equilibrium with a flow velocity $\langle V \rangle$, which we want to evaluate. The infinite gas–solid boundary coincides with the XOZ plane of the coordinate system, the OY axis with the external normal to the solid surface, and a plane SAW is propagating in the direction of the positive OX axis (see Fig. 1). The attenuation of the SAW is neglected.

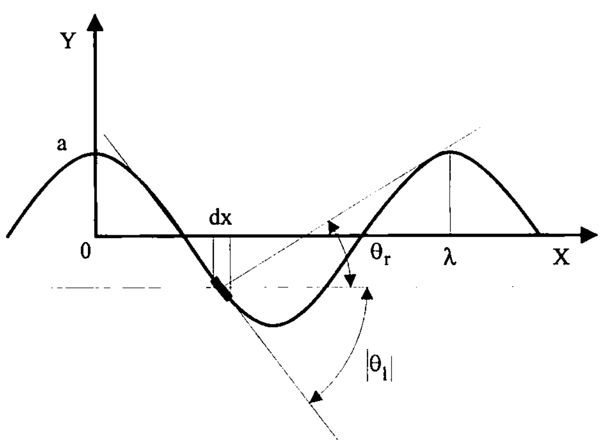

Fig. 1. The imposed restrictions on the angle of takeoff of the molecules from the deformed surface.

The mean density perturbation has to vanish, in view of the conservation of particles. This means that any nonuniformity in density Δn can yield a nonzero average contribution only via an interaction with another perturbation. The result would be of the order $(a/\lambda)^2$ or higher. Thus only a perturbation of the scattered molecular distribution can produce a mean effect of order (a/λ).

The surface scatters molecules with the distribution function

$$f_s(\mathbf{v}, \varphi) = [\langle n \rangle + \Delta n(\varphi)] \left(\frac{m}{2\pi kT}\right)^{3/2} \times \exp\left(-\frac{m}{2kT}\{[v_x - u_x(\varphi)]^2 + [v_y - u_y(\varphi)]^2 + v_z^2\}\right) \quad (1)$$

where $\varphi = 2\pi(x - Ct)/\lambda$ is the SAW phase at which the molecule leaves the solid surface, $\langle n \rangle$ the mean number density of molecules, $u_x(\varphi)$ and $u_y(\varphi)$ the longitudinal and the transverse displacement velocity components in the SAW, and C the SAW phase velocity.

The transverse displacement and displacement velocity in the SAW are

$$\Delta y(\varphi) = a\cos(\varphi), \qquad u_y(\varphi) = a\omega\sin(\varphi)$$

Molecules leaving the perturbed surface either leave the SAW or they collide again with the surface. What exactly happens is determined by the molecular velocity and by the angle of takeoff from the surface. The condition under which the molecule leaves the surface without repeated collision is

$$-\frac{v_y}{\alpha_l} < v_x - C < \frac{v_y}{\alpha_r} \quad (2)$$

where α_l and α_r are the tangents of the angles $|\theta_l|$ and θ_r that limit the exit trajectories on the left and on the right because of the deformation (see Fig. 1). The tangents are functions of the phase φ and satisfy the following relation:

$$\alpha_l(\varphi) = \alpha_r(2\pi - \varphi). \quad (3)$$

The function $\alpha_r(\varphi)$ for a sinusoidal surface is shown in Fig. 2.

Now, let us now calculate the mean tangential velocity V of the molecules leaving a surface element dx without the repeated collision:

$$V(\varphi) = \frac{1}{\langle n \rangle} \int_{(\mathbf{v})} d\mathbf{v}\, v_x f_s(\mathbf{v}, \varphi) \quad (4)$$

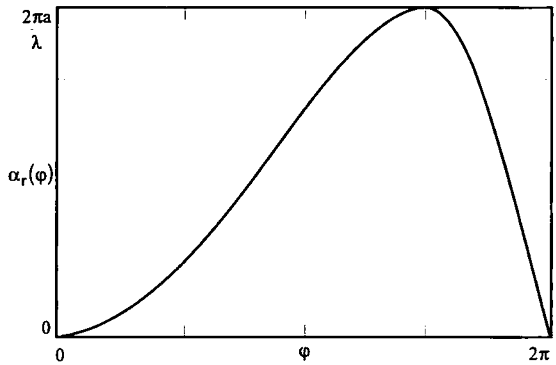

Fig. 2. The function $\alpha_r(\varphi)$ for a sinusoidal surface.

Using conditions (2) and Eq. (1), we obtain from (4) that

$$V(\varphi)=\frac{1}{\pi}\frac{\langle n\rangle+\Delta n(\varphi)}{\langle n\rangle}\left(\frac{2kT}{m}\right)^{1/2}\int_0^{\infty}dc_y\left(\exp\{-[c_y-u_y^*(\varphi)]^2\}\right.$$
$$\left.\times\int_A^B dc_x\,c_x\exp\{-[c_x-u_x^*(\varphi)]^2\}\right) \tag{5}$$

where

$$c_i=v_i\left(\frac{m}{2kT}\right)^{1/2},\qquad u_i^*=u_i\left(\frac{m}{2kT}\right)^{1/2},\qquad C^*=C\left(\frac{m}{2kT}\right)^{1/2},\qquad i=x,y$$

The integration limits for c_x follow from (2):

$$A=-\left(\frac{c_y}{\alpha_l(\varphi)}-C^*\right)\qquad\text{and}\qquad B=\left(\frac{c_y}{\alpha_r(\varphi)}+C^*\right)$$

Since u_x^* is small, the integral over c_x in (5) may be approximated as

$$\int_A^B dc_x\,c_x\exp\{-[c_x-u_x^*(\varphi)]^2\}$$
$$=\int_A^B dc_x\,c_x\exp\{-c_x^2\}+2u_x^*(\varphi)\int_{-\infty}^{+\infty}dc_x\,c_x^2\exp\{-c_x^2\}+o\left(\left(\frac{a}{\lambda}\right)^2\right)$$

Anticipating that the phase average of the second term on the right vanishes, we are going to delete it. The remaining integral from (5) reduces to

$$V(\varphi)=\frac{1}{2\pi}\frac{\langle n\rangle+\Delta n(\varphi)}{\langle n\rangle}\left(\frac{2kT}{m}\right)^{1/2}$$
$$\times\int_0^\infty dc_y\exp\{-[c_y-u_y^*(\varphi)]^2\}(\exp\{-A^2\}-\exp\{-B^2\})$$

Since $\alpha_i \leqslant 2\pi a/\lambda$ and $u_y \leqslant a\omega = 2\pi aC/\lambda$, the exponent in the first factor of the integral is smaller than that in the second at least by the ratio $(2\pi a/\lambda)^2$, which is very small number. Consequently, as long as the second factor is not vanishingly small, the first exponential can be replaced by 1.

The final result of integration is then

$$V(\varphi)=\frac{a\omega}{4\sqrt{\pi}\,C^*}\{[\alpha_r^*(\varphi)+\alpha_l^*(\varphi)]\operatorname{erf}(C^*)$$
$$-[\alpha_r^*(\varphi)-\alpha_l^*(\varphi)]\}+o\left(\left(\frac{a}{\lambda}\right)^2\right) \tag{6}$$

where $\alpha_i^*(\varphi)=(C/a\omega)\alpha_i(\varphi)$, $i=l, r$. Averaging of (6) over all possible φ, taking (3) into account, gives the mean velocity of the gas molecules leaving the disturbed surface,

$$\langle V\rangle=\frac{a\omega}{2\sqrt{\pi}}\langle\alpha_r^*(\varphi)\rangle\frac{\operatorname{erf}(C^*)}{C^*}+o\left(\left(\frac{a}{\lambda}\right)^2\right) \tag{7}$$

where $\langle\alpha_r^*(\varphi)\rangle=0.52$ for a sinusoidal surface.

It should be noticed that the lower bound of the integral (5) is not exact. Instead of 0 one should take $\max(u_y(\varphi), 0)=u_y(\varphi)\,\eta(\pi-\varphi)$, where η is the Heaviside function.

This leads to the more accurate result

$$\langle V\rangle=\frac{a\omega}{4\sqrt{\pi}\,C^*}\left\{\left\langle\alpha_r^*(\varphi)\operatorname{erf}\left(C^*\left[1-\frac{\sin(\varphi)}{\alpha_r^*(\varphi)}\eta(\pi-\varphi)\right]\right)\right\rangle\right.$$
$$\left.+\left\langle\alpha_l^*(\varphi)\operatorname{erf}\left(C^*\left[1+\frac{\sin(\varphi)}{\alpha_l^*(\varphi)}\eta(\pi-\varphi)\right]\right)\right\rangle\right\}+o\left(\left(\frac{a}{\lambda}\right)^2\right) \tag{8}$$

However, this is not very different from the more transparent function (7) (see Fig. 3).

Thus, the mean tangential velocity $\langle V\rangle$ of scattered molecules is of the order (a/λ). In the case of a semiinfinite gas environment, the steady

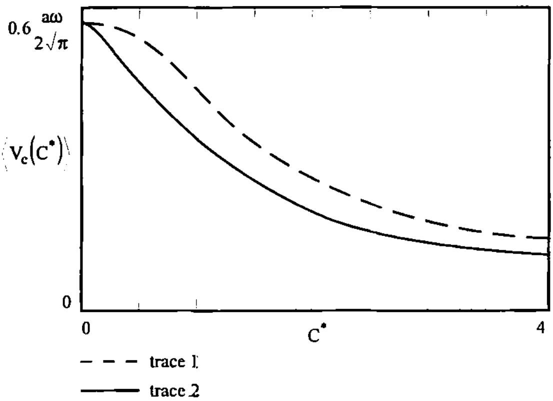

Fig. 3. The gas entrainment velocity dependence on the dimensionless SAW phase velocity C^*. The dashed and solid curves plots of Eq. (7) and (8), respectively.

flow velocity will be $\langle V \rangle$. In the case of a flat channel with a free molecular regime with respect to the channel height and with only one surface carrying the SAW, the flow velocity should be equal to $\langle V \rangle/2$.

Three factors may be separated in (7). The first shows that $\langle V \rangle \propto (a\omega)$. This is essentially different from the continuum case and also from earlier theories for the free molecular regime. The molecules must have a rather long free path $l \geqslant \lambda$, and move almost parallel to the initial nondisturbed surface "to feel" the surface deformation. This may explain the absence of such an effect in the continuum regime, where the surface seems to be effectively flat because the molecules collide with each other earlier than with neighboring areas of the deformed surface.

The second factor in (7) may be called a "form factor," since it takes into account the shape of the deformed surface.

The third factor in (7) describes the dependence upon gas parameters. The entrainment velocity depends on the temperature of the gas and upon its molecular mass. The higher the temperature, and thereby the thermal velocity of gas molecules, the stronger is the effect and vice versa: the greater the molecular mass, the less is $\langle V \rangle$. The mass dependence is the important distinction of the present model from earlier ones.[2,3]

The value of $\langle V \rangle$ will be about 0.1 m/sec for helium and 0.02 m/sec for xenon with $\omega = 10^9$ Hz, $a = 10^{-9}$ m, $T = 300$ K, and $C = 3500$ m/sec.

In conclusion, we would like to stress the need for an experimental investigation of the acoustic slip phenomenon, since it seems to be an attractive way of measuring SAW characteristics as compared to the laser method. The phenomenon may be observed by measuring the pressure difference in a plane closed channel.[1] The experiment could also decide whether or not the suggested model is valid.

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